PROBABILITY THEORY

Summary:

1. *Probability theory deals with events of chances.*

2. *An event is a single result of an experiment*

3. *A set of all possible outcomes of an experiment is called a sample space or possibility space*

4. *Probability of an event* = $\frac{Number\ of\ desired\ outcomes}{Number\ of\ possible\ outcomes}$.

5. *Probabilities can be expressed as fractions, decimals or percentages*

6. (i) *The probability of an event lies between* ***0*** *and* ***1***

(ii) *The probability of an impossible event is* ***0***

(iii) *The probability of a sure event is* ***1***

7. (i) *The probability of event* ***A*** *occurring is written as* ***P(A).***

(ii) *The probability of event* ***A*** *not occurring is written as* $P(\overline{A})$ *or* $P(A^{/})$ ***.***

(iii) *For any event* ***A,*** $P(A) + P(\overline{A}) = 1$ *or* $P(\overline{A}) = 1 - P(A)$

EXAMPLES:

1. *An integer between* ***1*** *and* ***10*** *inclusive is chosen at random****.*** *Find the probability that the chosen integer is****:***

(i) *more than* ***4***

(ii) *an even number*

(iii) *a multiple of* ***3***

(iv) *a triangular number*

2. *A box contains* ***3*** *red,* ***5*** *green and* ***12*** *blue pens. Find the probability that the pen picked from the box is:*

(i) *red*

(ii) *green*

(iii) *blue*

(iv) *not green*

(v) *not blue*

(vi) *either red or blue*

3. *A box contains red, green and blue pens. The probability of picking a blue pen from the box is* $\frac{4}{9}$ *and that of a green pen is* $\frac{1}{3}$. *Find the probability of picking a red pen*

4. *A box contains* ***8*** *red pens and the rest are blue. The probability of picking a blue pen from the box is* $\frac{3}{5}$. *Find the number of blue pens in the box*

5. *A fair coin is thrown once.*

(a) *Write down its sample space*

(b) *Find the probability of obtaining:*

(i) *a head*

(ii) *a tail*

6. *Two fair coins are tossed together.*

(a) *Write down its sample space*

(b) *Find the probability of obtaining:*

(i) *two heads*

(ii) *a head and a tail*

7. *Three fair coins are tossed simultaneously.*

(a) *Write down its sample space*

(b) *Find the probability of obtaining:*

(i) *exactly two heads*

(ii) *at least two heads*

8. *A fair dice is thrown once.*

(a) *Write down its sample space*

(b) *Find the probability of obtaining:*

(i) *an even number*

(ii) *a square number*

(iii) *either a **3** or a **4***

9. *Two fair dice are thrown together.*

(a) *Draw a table for the possible outcomes*

(b) *Find the probability that:*

(i) *both show even numbers*

(ii) *both show similar faces*

(iii) *one shows an even number and the other odd*

(iv) *the sum of the scores is **9***

10. *Two fair dice are thrown simultaneously. Find the probability that the sum of the scores is*

(i) *five*

(ii) *less than five*

(iii) *greater than six*

11. *A fair coin and a fair die are thrown together. Find the probability of obtaining a tail and an even number*

12. *Two fair coins and a fair die are thrown together. Find the probability of obtaining:*

(i) *two heads and a number greater than* ***3***

(ii) *at least one tail and a number less than* ***4***

13. *A two digit number is formed using the digits* ***2, 3*** *and* ***4*** *without repeating any digit in the number formed.*

(a) *Write down the possibility space for the numbers formed*

(b) *Find the probability that the number formed is:*

(i) *even*

(ii) *less than* ***30***

14. *A two digit number is formed using the digits* ***0, 3*** *and* ***5*** *without repeating any digit in the number formed.*

(i) *Write down the possibility space for the numbers formed*

(ii) *Find the probability that the number formed is less than* ***53***

Soln:

Hint: ***03*** *and* ***05*** *are non−two digit numbers since* ***03*** *is the same as* ***3*** *and* ***05*** *is* ***5***

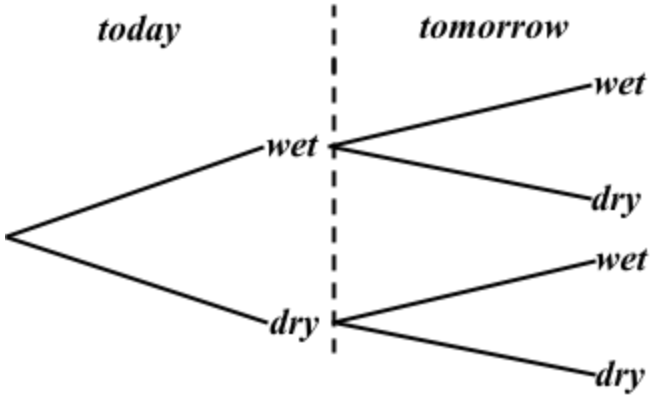
15. *A three digit number is formed using the digits* ***3, 4*** *and* ***5*** *without repeating any digit in the number formed.*

(i) *Write down the possibility space for the numbers formed*

(ii) *Find the probability that the number formed is even*

16. *The weather forecast says the probability that it will be wet today is* $\frac{3}{4}$ *and that of tomorrow is* $\frac{4}{5}$.

(i) *Copy and complete the probability tree diagram below:*



(ii) *Find the probability that it will be wet on just one of the two days*

17. *A coin is biased so that the probability of tossing a head is* $\frac{2}{3}$. *If it is tossed three times,*

(a) *Draw a tree diagram showing the possible outcomes*

(b) *Find the probability of obtaining:*

(i) *exactly two heads*

(ii) *at least two tails*

(iii) *two tails on the first two tosses*

18. *A box contains **5** red and **3** blue pens. Two pens are drawn in succession at random from the box with replacement.*

(a) *Draw a tree diagram showing the possible outcomes*

(b) *Find the probability of picking:*

(i) *pens of the same colour*

(ii) *exactly one red pen*

(iii) *at least one red pen*

(iv) *at most one blue pen*

19. *A box contains **8** red and **3** blue pens. Two pens are drawn in succession at random from the box without replacement.*

(a) *Draw a tree diagram showing the possible outcomes*

(b) *Find the probability of picking:*

(i) *pens of different colours*

(ii) *exactly one red pen*

(iii) *at least one red pen*

(iv) *at most one blue pen*

20. *A bag contains **5** red, **7** blue and **8** green beads. Three beads are drawn in succession at random from the bag without replacement.*

(a) *Draw a tree diagram showing the possible outcomes*

(b) *Find the probability of picking:*

(i) *three green beads*

(ii) *one bead of each colour*

(iii) *beads of the same colour*

21. *A bag contains **30** white, **20** blue and **20** red balls. Three balls are drawn in succession at random without replacement. Find the probability that the first ball and third ball is white.*

EER:

1. *An integer between **10** and **30** inclusive is chosen at random. Find the probability that the chosen integer is:*

(i) *prime*

(ii) *divisible by **2, 3** or **5***

(iii) *a triangular number*

(iv) *a factor of **240***

2. *A letter is chosen from the word **"MISCELLANEOUS".** Find the probability that it will be:*

(i) *an **S***

(ii) *a vowel*

(iii) *a consonant*

3. *In a game, a player tosses a fair coin once. When a **1** or **6** appears the player wins and when a **3** or **4** appears the player losses. Find the probability that the player neither wins nor losses.*

4. *A two digit number is formed using the digits* ***1, 2, 4, 5*** *and* ***7*** *without repeating any digit in the number formed.*

(a) *Write down the possibility space for the numbers formed*

(b) *Find the probability that the number formed is:*

(i) *divisible by* ***5*** ***(ii)*** *divisible by* ***2*** *or* ***5*** ***(iii)*** *greater than* ***50***

5. *Two fair dice are thrown together. Find the probability that the product of the scores is*

(i) *twelve*

(ii) *four*

6. *A fair coin is tossed three times.*

(a) *Draw a tree diagram showing the possible outcomes*

(b) *Find the probability of obtaining:*

(i) *three heads*

(ii) *exactly two heads*

(iii) *at least two tails*

7. *Two fair dice are thrown together. The score is the positive difference of the two numbers on which the dice land.*

(a) *Construct a table showing the possible outcomes*

(b) *Find the probability that the positive difference of the scores is*

(i) *two* ***(ii)*** *five*

8. *A box contains* ***15*** *red pens and the rest are blue. The probability of picking a red pen from the box is* $\frac{3}{5}$. *Find the:*

(i) *probability of picking a blue pen*

(ii) *number of pens in the box*

9. *A bag contains* ***3*** *white and* ***5*** *black balls. If two balls are picked in succession at random from it, find the probability that the second ball picked is white if picking is done:*

(i) *with replacement.*

(ii) *without replacement.*

10. *Three balls are drawn at random one after the other without replacement from a bag containing* ***4*** *white,* ***8*** *blue ,* ***5*** *red and* ***3*** *pink balls. Find the probability that the first ball is blue, the second red or blue and the third is white.*

11. *A box contains two types of balls* ***red*** *and* ***black****. When a ball is picked from the box, the probability that it is red is* $\frac{7}{12}$. *Two balls are selected at random from the box without replacement. Find the probability that:*

the second ball is black

12. *A box contains* ***3*** *red,* ***4*** *green, and* ***5*** *blue beads. Two beads are selected at random from the box without replacement. Find the probability of picking:*

(i) *beads of the same colour*

(ii) *at least one red bead*

(iii) *beads of different colours*

13. *A fair die and a fair octahedral die are thrown together and the outcome on each die is recorded.*

(a) *Draw a table showing the possible outcomes*

(b) *Find the probability that:*

(i) *both show odd numbers*

(ii) *both show similar faces*

(iii) *one shows an even number and the other odd*

(iv) *the sum of the scores is greater than or equal to* ***5***

(v) *the product of the scores is* ***4*** *or* ***12***

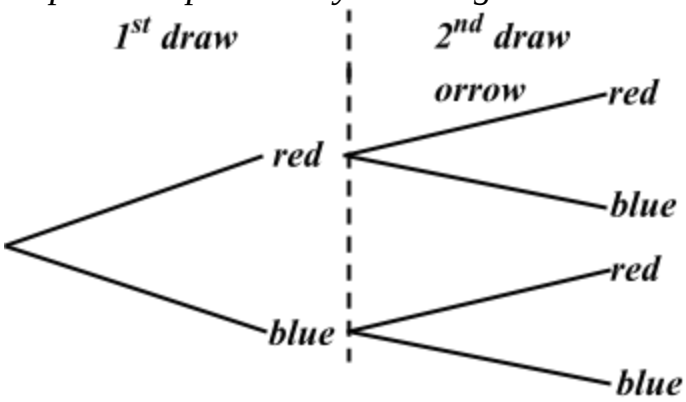
14. *A bag contains* ***x*** *red balls and* ***(x − 8)*** *blue ones. The probability of picking a red ball from the box is* ***0·75.*** *Find the:*

(i) *probability of picking a blue ball*

(ii) *number of balls in the bag*

15. *A box contains* ***7*** *red and* ***5*** *blue pens. Two pens are drawn in succession at random from the box without replacement.*

(i) *Copy and complete the probability tree diagram below:*



(ii) *Find the probability of picking at least one red pen*

16. *A fair die and a fair tetrahedral die are thrown together and the outcome on each die is recorded.*

(a) *Draw a table showing the possible outcomes*

(b) *Find the probability that:*

(i) *both show odd numbers*

(ii) *the sum of the scores is greater than or equal to* ***5***

(iii) *the product of the scores is* ***4*** *or* ***12***

17. *Two fair dice are thrown together.*

(a) *Draw a table for the possible outcomes*

(b) *Find the probability that:*

(i) *both show even numbers*

(ii) *both show similar faces*

(iii) *one shows an even number and the other odd*

(iv) *the sum of the scores is* ***9***

18. *Two fair tetrahedral dice each with faces numbered* ***1*** *to* ***4*** *are thrown together. The score is the sum of the two numbers on which the dice land.*

(a) *Construct a table showing the possible outcomes*

(b) *Find the probability that the sum of the scores is*

(i) *seven*

(ii) *less than six*

19. *Two fair dice are thrown together. Find the probability that the positive difference of the scores is*

(i) *two*

(ii) *five*

20. *A box contains **5** red and **3** blue balls. Two balls are drawn in succession at random from it without replacement. Find the probability that**:***

(i) *they are of the same colour*

(ii) *the second ball drawn is red.*

21. *A box contains **5** red and **3** blue balls. Three balls are selected in succession at random from it without replacement. Find the probability that**:***

(i) *they are of the same colour*

(ii) *the first and last are of the same colour*

(iii) *at most one blue ball is drawn.*

22. *Two fair tetrahedral dice are thrown up at once**.** Find the probability that the sum of the scores on the dice is less than **6.***